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Metanoia and the Market

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Abstract

If investors randomly switch between being rational and irrational, then eventually the market will be half rational and half irrational, even if all investors start off rational, no matter how low the switching probability is. Thus, mispricings can persist even with continued volume between two fundamentally identical investments. Multiple survey results for hypothetical investment scenarios support this metanoia model. In addition, the dynamics of a large market discrepancy in HSBC shares from 1992-1999 are consistent with metanoia. In short, the law of one price will be violated so long as there is any probability of switching: identical assets will have different prices.

Keywords: behavioral, law of one price, irrational, mispricing, arbitrage

JEL Classifications: G14, G10, G19

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I Introduction

If *Anna Karenina* were a finance textbook, Leo Tolstoy could have begun it, "Rational investors are all alike; every irrational investor is irrational in his own way."

It is hard to be consistently rational. Even if you choose rationally 95 percent of the time, then after just fourteen decisions, you are more likely than not to have made at least one irrational one.

The end result of ubiquitous rationality is the law of one price, one of the few laws that predate Adam Smith. It was discovered by the Scottish economist John Law in the early eighteenth century and can be best formulated as, "All identical goods must have only one price."

Ironically, Law's law may have first been violated by Law himself. Gleeson (2001), among others, describes his infamous tale. As chief banker, to finance expansion in Louisiana for France, Law issued paper shares that were guaranteed by the king and exchangeable into silver.

In 1719, they cost 500 livres. In 1720, they cost 10,000 livres and traded as high as 15,000 livres. Law coined the word "millionaire" to describe his investors. In 1721, the shares collapsed to their intrinsic value of about 300 livres, a loss of 98 percent. Law was exiled and spent the rest of his life in misery.

The law of one price (LOOP) is often associated with purchasing power parity (PPP). As Taylor and Taylor (2004) put it, PPP is "a disarmingly simple theory that holds that... a unit of currency of one country will have the same purchasing power in a foreign country." The PPP is almost as old as the LOOP and in its long history, is "flagrantly and systematically violated by empirical data" (Isard 1977).

However, the LOOP with respect to financial instruments other than currencies, for example in the realm of stocks, does not have as much data, primarily because there is much less historical stock market data. But we do know a few things. As e.g. Froot and Dabora (1999) and Lamont and Thaler (2003) document, the LOOP in financial securities is also sometimes violated, opening the door for any of a variety of cognitive and psychological biases to explain the various mispricings. Maymin (2007) describes a discrepancy in two classes of HSBC shares, both trading in the same currency, on the same exchange, at the same time; both members of the same indices; both having large trading volume; and yet having different prices, sometimes by as much as eight percent. Furthermore, the discrepancy changed signs several times: the typically cheap share class was sometimes expensive, and vice versa.

When a LOOP hole is found, it is often not difficult to explain it using any of a litany of psychological biases. However, this is the first paper to suggest that merely random switching will lead to persistent mispricing.

Who would do the random switching, and why? The answer of what the “metanoia” model is that each market participant randomly transforms from one type of investor into another. The randomness assumption proposes that there is no reason for the switch, but that the results of the switch persist. This assumption is counterintuitive but nevertheless the predictions appear to bear out in multiple survey results. This metanoia bears a surface similarity to the “coherent arbitrariness” of Ariely, Loewenstein, and Prelec (2003), who find in experiments that subjects price different consumption goods (specifically noises) coherently relative to each otherm but arbitrarily in absolute terms, giving too much weight to arbitrary anchors. In our case of financial goods, even the relative prices between assets are mispriced.

Standard behavioral finance rests on two assumptions: that there are limits to arbitrage preventing rational investors from eliminating a pricing discrepancy, and that there are systematic biases in irrationality (c.f. Barberis and Thaler (2003)). In this paper, I propose an alternative to the second condition, namely that investors, including arbitrageurs, randomly and independently switch between rational and irrational decision-making with some arbitrarily low probability. I then show that this leads to an equilibrium of an even split of rational and irrational investors, no matter how low the switching probability; thus, an arbitrary mispricing can persist indefinitely. I show survey results that support this metanoia model and discuss the HSBC discrepancy described in Maymin (2007) as a real-world example.

Hence, not only is the law of one price wrong, but people will continue trading at arbitrary values for identical shares so long as our single assumption of independent random metanoia holds.

II Metanoia

Metanoia (from the Greek word meaning changing one's mind, or repentance) is commonly defined as a transformative change of heart, especially a spiritual conversion. It is more permanent than a whimsical decision but less justified than a logical one. In Carl Jung's psychology, it is a term indicating a spontaneous reorganization of the psyche to heal itself of unbearable conflict. The way we use the term here will be to suggest a shift in a person from being *rational* to *irrational*, where those terms are themselves defined based on whether the person tends to buy the cheaper or more expensive of two identical securities, respectively.

What does it mean for securities to be identical? Obviously they are not identical in all respects: one of them is presumed to be cheaper than the other, so they must differ at least on price. So let us suppose there are two *fundamentally identical* securities available, trading at different prices.

As an example, think of dual class shares such as Royal Dutch-Shell, which have two shares traded on different stock exchanges but each representing an equal share in the company. The Royal Dutch-Shell discrepancy is described in Froot and Dabora (1999).

An even better example is HSBC, described in Maymin (2007), which had two share classes from 1992 to 1999. The two share classes had exactly identical dividends, taxation, and voting rights. They traded in the same currency on the same exchange at the same time. They were both members of the major indices and they were two of the most highly traded shares in all of London. Yet one often traded at a higher price than the other. What's more, it wasn't always the same one: each share class spent time being overpriced relative to the other one.

How can these two otherwise identical securities have different prices? Does one of them proxy for some kind of higher quality or other intangible? The empirical answer from the unique case of HSBC is a resounding no, primarily because the premium changed sign: the expensive share was at times the cheap one, and the cheap one the expensive. In short, the market seems to be stuck in its own circular logic: the two securities are different because they have different prices, and they have different prices because they are different. The surveys we run make the definition particularly sharp.

Why do these different share classes exist at all? In the case of HSBC, it was because HSBC, a Hong Kong-based bank, purchased Midland Bank. HSBC already had shares listed on the London Stock Exchange that were exchangeable into the Hong Kong shares, but they had a par value denominated in Hong Kong dollars. The par value does not affect the trading value or the tax implications, and is merely a stamp that is put on the share certificate, but HSBC could not purchase Midland Bank with those London shares, so they created a new share class intended to be in every way identical with the first, with the only difference being that the declared par value was in British pounds. The listing particulars of the new share classes made it abundantly clear that the two share classes would be treated, for *all* intents and purposes, as equal.

Given an initial arbitrary difference in price, which of the two share classes should investors buy? If all investors are rational and the LOOP holds, all investors should buy the cheaper share, and so we will never observe a trade of the more expensive share. Indeed, there will never be a more expensive share. But that is not what happens in reality.

By the metanoia model, on the first day, investors randomly choose which of the two securities to invest in. Some people (let's say one in four for concreteness) buy the more expensive one. Arbitrageurs are not exempt from metanoia and so the mispricing can persist.

Let's call those that buy the cheaper of two identical securities "rational" and those who buy the more expensive one "irrational."

This choice of labeling does not mean that the investors we call irrational are stupid or confused. On the contrary, as we will see from their thought processes, they come up with just as many logical reasons for buying the expensive share as the rational ones do for buying the cheap share. There is no single reason that causes one group to buy the expensive one and the other the cheap one: in fact, by our metanoia model, people randomly switch from one type to the other, and so adopt new reasons.

Nevertheless, there is a clear difference between decisions we call rational and those we call irrational. One might argue that those who buy the more expensive security are doing so rationally based on the following thought process: the more expensive one is probably more expensive for a reason, a reason that others do not know; therefore I should buy it. However, this logic is fundamentally irrational because an investor accepts his ignorance of the reason behind the mispricing, takes the mispricing as evidence that there ought to be a reason, and concludes that he is now not only no longer ignorant, but more informed than the market, and that the expensive price should be even more expensive.

In both the survey and in most mispricings in reality, the only fully rational response is to always buy the cheaper share. We do not, however, need to specify what is the precise type of irrationality. As can be seen in Appendix B, people rationalize all sorts of irrational decisions.

Furthermore, the type of person each investor is, rational or irrational, persists. If you are rational today, you will likely be rational tomorrow. However, you have some probability of switching to the other track.

This model of individual switching distinguishes from Bikhchandani, Hirshleifer, and Welch (1992) because their model of information cascades applies to sequences of signals evaluated by different agents. Metanoia is a model of individual choice that does not rely on the choices of others and which always leads to a single equilibrium of equal amounts of rational and irrational investors.

After the investors choose between the two securities on the first day, the investors must choose again the next day, or perhaps the next month. One quarter of those who were rational last time will switch and become irrational, and one quarter of those who were irrational last time will switch and become rational.

It is important to note that investors do not roll the dice of irrationality every single day or every single hour. Something must appear significantly different about the situation so that they approach it as a new decision to be made, yet it must be sufficiently recognizable as belonging to the same domain so that they recall whether their previous decision was rational or not. A concrete example would be the receipt of seemingly new information about an existing trade.

The metanoia model presented here is the simplest possible: it assumes a single, constant, independent probability that is identical for switching from rational to irrational and from irrational to rational.

III Survey Results

I ran a survey on three distinct groups, asking them which of two fundamentally identical but differently priced items they would buy under a variety of seemingly different circumstances. The three groups were summer students at the University of Chicago (59 respondents),

subscribers to the "Friends of Positive Psychology" email list run by the American Psychological Association (88 respondents), and members of the Harvard Startups group on Yahoo (77 respondents).

A consistently rational person would always buy the cheaper of two identical items. Of the 224 total respondents, only 71, or approximately one third, responded in this fashion. If we restrict it to the realm of financial securities only (for reasons described below), then only 75 people responded completely rationally.

By the metanoia model, the probability of responding rationally four times in a row is $(1-p)^4$, where p is the probability of switching. If we solve for p in:

$$(1-p)^4 = \frac{75}{224}$$

then we see that the calibrated switching probability is $p = 23.9\%$, which is why we can roughly say that about one in four people switch between rational and irrational responses each time they make a repeat decision in the same domain.

The survey begins with a choice between two common commodities, rather than investments, which will allow us to test two other predictions of the metanoia model by comparing the responses between consumption and investment goods.

We have defined it to be "irrational" to buy a more expensive financial security when a cheaper and otherwise identical alternative is available. But what about potatoes? Suppose for the moment that the choice is between two otherwise identical consumable goods. Now the decision is not about the direction of future price appreciation, but taste and consumption. In this case, intangibles such as quality can enter into people's thinking. So what would people do?

The results in brackets below are the sum across all 224 responses[1].

> **Question 1.** You are going to the market to buy potatoes for your sick relative in order to make him a good soup. At the market, two twin brothers, A and B, are selling potatoes that are from the same garden and of the same quality. Potato experts are unable to distinguish the two.
> But brother B's potatoes are priced a little higher than brother A's. About the same number of people buy potatoes from brother A as from brother B.
> You need to purchase one pound, either from A for $1 per pound or from B for $1.02 per pound. Which do you buy?
> Buy from brother A for $1 [208, or 93%]
> Buy from brother B for $1.02 [16, or 7%]

Virtually everybody chose to buy the cheaper potato. The metanoia model predicts one in four will be irrational; instead it is one in twelve. Though a more complicated model could of course fit more of the survey response data, we are most interested in the broad predictions, which are the same even for the minimal metanoia model with its single parameter.

Now, would it make a difference if financial securities were involved instead? Metanoia predicts two things: 1) that approximately the same number of people will respond irrationally as in the first question, and 2) that the irrational respondents will be different people. In particular,

[1] There are two ways to combine the three surveys. One is to simply sum across the number of responses as if they were all respondents to the same survey; that is the approach I take in the text. The other is to take averages of the percentage responses of each group. Because the number of respondents in each group is roughly similar, there is no significant difference between the two approaches: the percentages match to within one or two percentage points.

very few of those who bought the more expensive potato would also buy the more expensive share.

Why? Because the two questions are in different domains. (The remaining questions in the survey stay in the domain of financial instruments, so changes in responses will be relative to the previous responses, as opposed to the blank slate starting point of complete rationality that occurs in different domains.)

> **Question 2.** You have decided to purchase one share of a company XYZ. The company has two share classes, A and B. Each share class is entitled to the same dividends, the same voting rights, and the same tax treatment. Both are members of the major indices and both trade the same volume every day. Analysts report that the two classes should trade at the same price.
> But the B class shares are more expensive than the A class. About the same number of people buy shares A as shares B.
> You need to purchase one share, either A for $100 or B for $102. Which do you buy?
>
Buy A share for $100	[208, or 93%]
Buy B share for $102	[16, or 7%]

Indeed, it turns out to make no difference in overall rational response rates. About one in twelve people, for varying reasons, would choose to buy the more expensive of two seemingly identical items, and that proportion seems to be about the same both for consumable goods and financial securities. Furthermore, again as predicted by the metanoia model, it is not an imbued characteristic of the individual person that they always buy the more expensive product: only five out of the twenty-seven respondents who bought either the more expensive potato or the more expensive share would have bought both. Rather, it appears to be more as if a rational person has a one in twelve chance of picking the more expensive of two options at any given point in time. These results conform with the metanoia model.

What would happen if the cheaper share stayed cheap while the more expensive share got even more expensive? The metanoia model predicts two things: 1) that about a quarter of those who responded rationally in question 2 will now respond irrationally, and 2) that about a quarter of those who responded irrationally in question 2 will now respond rationally.

> **Question 3.** Suppose you bought the cheaper A share. After a few months, the A share is still worth $100 but the B share is now worth $104. You sell out of your position with neither a profit nor a loss. Then another few months later, with prices unchanged from their new levels, you need to purchase one share of the company again.
> You need to purchase one share, either A for $100 or B for $104. Which do you buy?
>
Buy one A share for $100	[144, or 64%]
Buy one B share for $104	[80, or 36%]

Assuming an earlier decision to purchase the cheap share, how do people react when their values are questioned? Their investment has neither made nor lost money but the other share has become even more expensive. How do they respond? They are even more anxious to buy the more expensive share now, with five times the irrational response rate.

By the metanoia model, of the 208 rational respondents of Question 2, one quarter, or 52 people, will switch to become irrational, and of the 16 irrational responses, one-quarter, or 4, will

switch to become rational. That predicts 208 - 52 + 4 or 160 rational responses. The actual response is 144.

> **Question 4.** Please select true if you agree with this statement or false if you disagree.
> If the B shares traded as high as $110 then came back down to $102, while A shares were unchanged at $100, then if I need to buy a share of the company, I would buy the B share for $102 instead of the A share for $100.
> False [114, or 51%]
> True [110, or 49%]

Another increase in the irrational response, just from the additional (but presumably irrelevant) knowledge that the discrepancy was even wider in the past than it is now.

Metanoia would predict that three quarters of the 144 rational people from Question 3 would remain rational, or 108, plus an additional one quarter of the 80 irrational ones, or 20, would switch, for a total of 128. The actual response was 114.

Already about half of the respondents are quite happy to buy the more expensive of the two otherwise seemingly identical shares. What if the expensive share had risen to even greater levels in the past? Would that affect their decision?

> **Question 5.** Please fill in the blank with the minimum number that would reflect what you would do. If there is no number that would make this statement true for you, please write "No Such Number" or "NA".
> If the B shares traded as high as $___ then came back down to $102, while A shares were unchanged at $100, then if I need to buy a share of the company, I would buy the B share for $102 instead of the A share for $100.
> No Such Number or NA [112, or 50%]
> The Number $*X* [112, or 50%]

By the same logic as above, the metanoia model would predict $\frac{3}{4}114+\frac{1}{4}110=113$ rational responses. The actual response was 112. Exactly half of the respondents are rational and half irrational.

The responses that gave a number $*X* had an average value of $116.9 and a median value of $110.

Since a 50% split means that the current discrepancy is essentially at equilibrium, this survey suggests that a pricing discrepancy between two otherwise seemingly identical securities would persist if it had been wider in the past. In other words, there is a discrepancy because there was a discrepancy.

Unequal splits mean that the current discrepancy is not at equilibrium because more investors prefer one of the two shares to the other at the given levels. Specifically, in the early survey questions, the cheaper share had a higher share of respondents. In the case of HSBC, this would suggest that the cheaper share would quickly be heavily purchased by investors. Indeed, in the early days of the HSBC discrepancy as described by Maymin (2007), the new share class was initially cheaper, a mispricing which essentially disappeared quite quickly, within a week, before gradually returning for longer durations. Thus, the excess purchasing of the cheaper class by metanoia investors could mitigate the mispricing. However, there are factors that prevent this natural mechanism from always reaching equilibrium: first, there is random noise in the mispricing beyond the demand of the buyers; second, there are additional impacts from possibly static supply since short selling is not a commonly employed tactic by individual investors; and

third, the survey asks respondents to assume they must purchase a share of the new company, while in reality, this choice is endogenous, and need not be constant over time. Thus, because of these restrictive factors, should the discrepancy happen to last long enough, it could in principle last forever as metanoia investors reach their eventual equilibrium because they have become acclimated to whatever the mispricing level happened to be.

Kluger and Wyatt (2005) show that markets reflect irrationality only if there are fewer than two rational arbitrageurs capable of pushing prices to their correct value. In our example, limits to arbitrage are presumed to prevent arbitrageurs from fully exploiting the mispricing, thus leaving the law of one price as the last resort for correctly pricing securities. And we have seen that the law of one price will be violated in equilibrium so long as investors experience metanoia, a consistent switching with low probability.

In our example, even though we began with all rational investors, a simple and small probability of switching led quickly to an equal ratio of rational and irrational investors.

Figure I shows a sorted graph that plots a black square when the respondent answered the survey question rationally (by buying the cheaper share) and a white square otherwise. The first column plots 224 black squares under the assumption that all respondents were initially rational. Note how an approximately constant proportion of respondents switch to irrationality for each question; note also how a similar proportion seem to randomly switch back. The sixteen irrational responses of question 1 are represented by the white boxes at the bottom of the second column; all but two of those irrational responders change their minds at least once more over the subsequent three more questions.

What made people change their minds across the different questions? Very little actual information. Since nothing was stated about the direction of the market or any other potential risk factors during this time, the changes in the responses cannot be due to a calculation of risk loadings. The only information provided was essentially a description of past price movements.

By the efficient markets hypothesis (c.f. Fama (1998)), such past information should already be reflected in the current prices. The only question that provided economically different information was Question 3, which stated that the more expensive share got even more expensive. Yet even there, people were only *more likely* to buy the expensive share, not less likely.

The number one-quarter is not magical. We can see that any non-zero probability of switching from rational to irrational or back again will eventually result in an even split between rational and irrational people, even if we start with all rational people. How? At each step, one of the groups is larger than half the population. At the next step, more will switch out of that group than will switch back, because the other group is smaller. So the majority becomes smaller and the minority becomes larger until an equal split is reached.

Appendix A provides the technical details by formalizing the switching assumption and showing that the limit is half rational, half irrational regardless of the switching probability or the initial conditions.

These survey responses were not automatic or knee-jerk by any means. Optional comments revealed the internal agony of some of the respondents. Tolstoy would have noted that all rational responses were identical: just buy the cheaper share, or potato. But the irrational choices were varied and individualized: "more volatility," "information available that I am not aware of," "B shares were being manipulated on a temporary basis," or "gut feel and heuristics." Appendix B lists more sample respondent comments.

IV Conclusion

How can two fundamentally identical securities trade at different prices if all investors start out rational? Traditional behavioral finance assumes that some portion of the population of investors is irrational and their biases are systemic so the market does not wash out the differences. This paper presented an alternative hypothesis that people have some probability of switching from being rational to being irrational, and back again. This metanoia model predicts that for any such probability, no matter how small, there will eventually be an even split of rational and irrational investors. Furthermore, after such an equilibrium is reached, a portion of them continue to switch and thus continue to trade with each other. Therefore, this model predicts that two identical securities can have different prices and continue to have arbitrarily large volume at those prices.

Furthermore, this model distinguishes from merely errors in judgment or mistakes. Errors are independent and a rational person who erroneously chose the more expensive share last time will still be more likely to choose the cheaper share this time; however, our survey results show that people tend to switch completely. If they chose irrationally, they are now more likely to choose irrationally again, unless they experience another rare metanoia. In addition, the sample comments provided show that the choices are not mistakes but the result of coherent thought and argument, whether they are purchasing the discounted or the premium share.

The implications of this model are not limited to finance. Consider political parties. If there are two major parties, as there are in America, then even if virtually all voters start out Republicans, so long as there is a non-zero probability of switching parties, the split will quickly grow closer to fifty-fifty with the Democrats. As above, this is true even if one of the parties proposes higher taxes or more spending than the other: the prices of the securities or parties does not matter.

Future research could include running the same or similar surveys on more and different kinds of participants, changing the questions to standardize the steps between each one so that the switching model could be checked with more precision, and checking what happens if the price difference between identical securities or commodities grow from a few percent to a few hundred or a few thousand percent. The model could also be extended, such as incorporating a different probability for switching from rational to irrational than from switching back, incorporating inertia so that switching back can only happen a given number of time steps after the initial switch, or allowing for more than two securities.

Finally, explicit trading could be introduced in an experimental market setting to evaluate the degree to which a market mechanism would inhibit or exacerbate, if any, the resulting equilibrium.

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A Technical Details

Investors are either in the rational or the irrational state and they switch from one to the other with probability p and remain in their existing state with probability $1-p$. This describes a regular Markov chain with the following transition matrix:

$$M \equiv \begin{pmatrix} 1-p & p \\ p & 1-p \end{pmatrix}$$

The stationary distribution $\pi \equiv (a \quad 1-a)$ must satisfy $\pi M = \pi$. Expanding the left hand side gives:

$$(a \quad 1-a)\begin{pmatrix} 1-p & p \\ p & 1-p \end{pmatrix} = (a - 2ap + p \quad 2ap + 1 - a - p)$$

Setting this equal to $(a \quad 1-a)$ has only one solution for a, namely $a = \frac{1}{2}$, and so the stationary distribution is:

$$\pi = \left(\frac{1}{2} \quad \frac{1}{2}\right)$$

Because the transition matrix M is irreducible (it is possible to get from any state to any other state) and aperiodic (it is possible to remain in each state), the Markov chain converges (c.f. Grinstead and Snell (1977)) to this unique stationary distribution regardless of where it begins.

B Sample Survey Responses

University of Chicago Summer Students

Respondent who gives fully rational responses to all questions: "I'm not sure what the history of pricing has to do with the current decision. [p]rospectively with the same rights as a B share, the A share is better value, regardless of the history of B - in fact the history of B suggests a bubble?"

Another respondent who gives fully rational responses to all questions: "Funny, (regarding company XYZ's different share classes) after reviewing my thought process I realized that I was making one judgment based upon the price and a separate judgment based upon the assumed risk. Originally, I immediately concluded that both shares represent the same asset; I then concluded that the A shares represented a better absolute value; I next concluded that the A shares also represented a better risk-adjusted value. However, I am now of the mind that if I believe the market's pricing activity is irrelevant of the true intrinsic (economic) value of the company then it should be equally irrelevant to the inherent risk tied to said value. That is to say, if I believe the two share classes represent identical underlying assets, then the price movement of one should not cloud my buying decision. As such, I am sticking with the better value in class A."

Another respondent who gives fully rational responses to all questions: "I am a GSB student. We are taught to take advantage of market irrationality by using arbitrage trades. This is how LTCM made money. Even though they got screwed, their arb trades would still have made money in the long run, they just needed more cash for their margin calls, the market was so out of whack. So buy share A and sell share B, and when they eventually converge, you have a profit. As for the potatoes, I do not really view it as an arb opportunity, but if they are identical, then I will save the two cents and also reward brother A for his consumer-friendly pricing. What are we supposed to do, buy the more expensive potatoes/shares because they have the aura of higher quality?"

Respondent who always buys the more expensive stock, but the cheaper potato: "Being close to graduation, it is a little embarrassing to admit that the public's pricing of a stock carries more sway than the analyst price target. I tend to stick with stock B, because I believe in the theory of market efficiency, and tend to think that there is some reason, something has been said or done that is visible only to a few people, and that has resulted in the higher price. Call it complete gut feel and heuristics, but if the price is only different by a couple of bucks, and there is a small indication that something `informal' is increasing the price, I'm willing to take the chance on B. With potatoes, I can judge a potato as good as anyone - ate a lot of potatoes growing up (didn't buy as many stocks)."

Respondent who buys the cheaper potato and share initially unless the premium first rises as high as 3% then comes back down: "I know that it shouldn't matter, but given a changing price I would wonder if there is information available that I am not aware of."

Respondent who buys the cheaper potato and share initially unless the premium first rises as high as 6% then comes back down: "Even though you stipulate the two stocks are identical, no one will believe they truly are. There is something that makes B's price change and we are left to assume you left some information out."

Respondent who gives fully irrational answers: "As I am somewhat risk averse, and without knowing anymore information about the stocks, I am a little more concerned about the more volatile stock. If it can go up 10%, it could go down 10% as well. However, for a stock to rise 20%, there would (hopefully) have to be enough forecast good news for it to increase that much that the future potential is greater."

Respondent who always buys the more expensive share but the cheaper potato: "For question 1 - I cannot see any reason to buy the more expensive potatoes. Brother B is just being greedy. Plus, the price I pay for potatoes today has little effect on me, since I cannot sell my potatoes back to the grocery store later. But for Questions 4 & 5 - $10 is a 10% difference. If it were any lower, I would just stick with A, since it seems more stable and does not fluctuate as much as B does. A 10% return is good enough for me."

Respondent who purchased cheaper potato and cheaper stock initially, but then bought when it got more expensive, and would buy again it if it rose to $110 and came back down, though lists minimum such threshold as $115: "I purchase stock for one of two reasons: For investing or for trading. I hold onto my investments for the long haul and make a concerted effort not to worry about short term drops in value, since I believe in the value of the company itself. For trading stocks, I'll watch more technical aspects, such as beta, trends, etc. I have a few of these, and have earned enough to pay for my GSB tuition (mostly luck, I admit). Thus, for this problem, I saw more volatility, with a fallback to a price close to the original. I've seen this happen with news announcements and other developments that give the stock a short-term spike. Case in point: EAG. I bought it after two such spikes, both of which were under .5 a share. I held onto it and sold at $2 a share. It's now back to .87. The company hasn't sold anything--the spikes resulted from news announcements and eager traders. I see more movement in Stock B--for a trading play like this (not investing), I'll go for momentum, since I'm looking to make a quick sale. Assuming it's got a base around $100, I'd go for it."[2]

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Respondent who gives fully rational responses to all questions: "Darn, I hope I haven't shown my ignorance."

Another respondent who gives fully rational responses to all questions: "I am a cheapskate and never buy higher priced items if I can help it. Your A/B share scenario proves my point - there is a lot of instability in B and that suggests higher risk."

Another respondent who gives fully rational responses to all questions: "This was a difficult question to answer honestly as one never really knows what will happen with a stock -- I wouldn't have knowledge that it would go up to anything, and it is unlikely that there would be no variability in stock A. Thus, I answered based on my own practice of buying for value and to hold rather than as speculation, which stock B questions seemed to imply."

Respondent who was rational for questions 1, 2, and 5, but not 3 and 4: "That's why my husband does this chore."

[2] The response was not an aberration. Five responses had the following seemingly inconsistent response: they would buy the more expensive share if it first went up to $110 then back to $102, but the "minimum" price the more expensive stock would have to rise to, before going back down to $102, such that they would then buy the more expensive stock, is higher than $110.

Respondent who bought the cheaper item for four straight questions but then turned irrational if the B share first rises to $125 then comes back: "It probably suggests I'm not a savvy investor, but I knew that."

Respondent who bought the cheaper item for four straight questions but then turned irrational if the B share first rises to $150 then comes back: "I'm very unlikely to engage in monetary speculation. It does not appeal to my value system, since I prefer to believe I can earn what I receive, rather than win something through clever gambling, which may also mean that someone else has to lose so I may gain."

Respondent who was rational for the first two questions: "I'm sure I made some irrational decisions in this - probably several - and now I will go ponder on what they were."

Respondent who was rational for all the questions except Question 4: "If I was shopping for shoes And one had a designer label but was the same quality of the non labeled pair and cost twice as much - I would pay twice as much for the label's intrinsic value."

Harvard Startups

Respondent who gives fully rational responses to all questions: "If the B share consistently traded higher than the a share and also had more volatility I would think that there is some difference between the two that I don't understand. That difference, whatever it is, must make B more valuable."

Another respondent who gives fully rational responses to all questions: "Interesting - I bet that the price going up will attract many - but no matter how high it goes, if it comes back down, i would view the drop as more significant."

Another respondent who gives fully rational responses to all questions: "I felt that the B shares were being manipulated on a temporary basis. (maybe boiler room brokers were pushing it, or they were receiving publicity, etc.) In the long term, the A shares would appreciate to the same price as the B shares, but for a higher gain."

Respondent who was rational for all the questions except Question 4: "I think the question hinges on whatever B's "brand" is - B has some special mojo that just doesn't come through in the questions."

Respondent who was rational for all the questions except Question 5: "I'd be assuming that the company with the higher climbing shares did something good from a marketing, etc. perspective to get buzz and that the other company would "get it" and do something too. For potatoes, why pay more for the same thing?"

Respondent who was rational for the first two questions: "Re: potato example, I do think under similar cirumstances for a manufactured or processed product, so not produce, I would skew for the more expensive option, i.e. the branded shampoo or toothpaste, for the "irrational rationale" that it must be better ...as a consumer I try to avoid this trap but at times to find myself more comfortable with brand over saving."

Figure I: Rational Responses. The plots below show a dark square when the respondent rationally answered the question number indicated by the column label, and a light square when they answered irrationally. The first column, labeled zero, plots dark squares under the assumption that all respondents were initially rational. The five panels are split based on which question the respondent first answered irrationally. Note how an approximately constant proportion of respondents switch to irrationality for each question, and how a similar proportion seem to randomly switch back.